A Solvable Model for Nonlinear Mean Field Dynamo

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We formulate a solvable model that describes generation and saturation of mean magnetic field in a dynamo with kinetic helicity, in the limit of large magnetic Prandtl number. This model is based on the assumption that the stochastic part of the velocity field is Gaussian and white in time (the Kazantsev-Kraichnan ensemble), while the regular part describing the back reaction of the magnetic field is chosen from balancing the viscous and Lorentz stresses in the MHD Navier-Stokes equation. The model provides an analytical explanation for previously obtained numerical results.

1. Turbulent motion of a conducting fluid generates magnetic fields in planets, stars, and interstellar medium. Observational [1] and theoretical [2–7] data suggest that simultaneously with developing intense small-scale fluctuations, magnetic fields appear to be ordered on scales much larger than the correlation length of the turbulent velocity field. Explanation of the large-scale field has remained a challenge for the astrophysical dynamo theory for a long time. A leading approach suggests that the large-scale field is generated due to broken parity in the turbulent velocity fluctuations. Mathematically, this means that the fluctuating velocity field u possesses nonzero helicity, i.e., $\int u\cdot(\nabla\times u) \neq 0$. The analytical confirmation of the large-scale field generation is based on the splitting of the magnetic field into the large scale and small scale components (compared to the velocity correlation length), and averaging the induction equation over the small scales [8]. This procedure leads to the equation for the mean magnetic field $\langle B^i\rangle$, which can be written in the general form:

$$\partial_t\langle B^i\rangle = \nabla\times(\alpha\,\langle B^i\rangle) + \beta\Delta\langle B^i\rangle + \dots \qquad (1)$$

Functions α and β should be obtained from the theory. As an estimate, one can accept $\alpha \sim u$ and $\beta \sim u\,l$, where l is the characteristic scale of the velocity field. It is assumed that the mean field is changing slowly in space, therefore the higher order space derivatives of $\langle B^i\rangle$ are omitted in (1). The crucial fact is that the coefficient α describing the amplification of the mean field is zero if the velocity fluctuations possess no helicity [8].

There is no general theory that would allow one to derive coefficients α and β from the microscopic equations. A self-consistent derivation of the mean field equation can be carried out under certain simplifying assumptions. One of them is the assumption about the geometry of the problem (velocity field is assumed to be two-dimensional), which leads to a solvable model introduced by Vainshtein [9]. Another assumption is about smallness of magnetic fluctuations compared to the mean field (see, e.g., [10,11]). However, analytical and numerical results on the initial (kinematic) stage of the dynamo demonstrate that it is the small scale magnetic fluctuations that are most energetic, not the large-scale mean field [5,12]. Therefore, an analytical treatment of this opposite limit in a general 3-dimensional case is in demand.

Moreover, the previous analytical and numerical investigations raised the question of the so-called catastrophic α-quenching in the mean-field dynamo. The mean magnetic field was shown to saturate when its energy reaches the equipartition value divided by the magnetic Reynolds number, R_m [11,13]. For the critical discussion of the α-quenching see also [14]. The magnetic resistivity thus played an essential role in the saturation mechanism. Astrophysical applications, e.g., galactic dynamos, provide a unique physical setting of extremely large magnetic Prandtl numbers (ratio of fluid viscosity to magnetic diffusivity), reaching 10^{14}–10^{22}, where the back reaction of the growing magnetic field can come into play *before* the magnetic field reaches the diffusive scales. The definitive numerical investigation of this problem requires very high resolution of the energy containing fine scales and is presently without the reach. An analytical understanding of this limit is therefore important.

In the present paper we introduce a solvable model describing the nonlinear mean field dynamo in the limit of a large magnetic Prandtl number. We do not resort to any artificial scale separation procedure or to the special geometry, and do not assume that the magnetic fluctuations are small. Instead, we make simplifying assumptions about the velocity field. We assume that the microscopic fluctuations of the velocity field are homogeneous, Gaussian, and short-time correlated, which constitutes the so-called Kazantsev-Kraichnan ensemble [4,3], while the the regular, "back reaction," part is chosen from balancing the viscous stress and the magnetic field stress. We also assume that the random velocity field possesses helicity. We believe that this model can provide some insight into understanding of the mean-field dynamo, and

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can be complimentary to the previously developed analytical treatments.

We show that in our model the fluctuations of the magnitude of the magnetic field are independent of the fluctuations of its direction. In the presence of helicity, the probability distribution function (PDF) of the direction of the magnetic field becomes nonstationary and anisotropic in x space. However, the probability distribution function of the magnitude of the magnetic field does not depend on x. Although the anisotropic part of the PDF decays in the course of time, the *magnitude* of the magnetic field increases. The interplay of these two factors leads to the generation of the non-decaying mean magnetic field, $\langle B^i \rangle$. This exact mechanism of the mean magnetic field generation is the first main result of the paper. The mean magnetic field is shown to be determined by the geometry of the problem and by the boundary conditions; the fact previously emphasized by Blackman and Field [15].

When the Lorentz force becomes strong enough, one cannot neglect its back reaction on the velocity field. If the velocity field helicity is small, this back reaction affects only the probability density function of the magnetic field magnitude, not the PDF of the magnetic field direction. As an example, we present a model based on the balance of viscous stress and magnetic stress, that can be solved exactly and clearly demonstrates this saturation: anisotropy of the PDF decays as before but the second factor, the magnetic field *magnitude*, saturates and cannot support the mean magnetic field anymore. The α term thus becomes reduced. This mechanism of saturation of the growing mean magnetic field is the second main result of the paper.

In Section 2 we formulate the model and proceed with the detailed derivation of the mean field equation. In Section 3 we suggest a simple closure describing dynamo saturation within the present model. Section 4 discusses possible generalizations of the model and its limitations.

2. Let us first neglect the Lorentz force and assume that the fluctuating velocity field $u^i(x,t)$ is random, homogeneous, Gaussian, and short-time correlated:

$$\langle u^i(x,t) u^k(x't')\rangle = \delta(t-t')\kappa^{ik}(x-x'), \tag{2}$$

where due to the lack of mirror invariance we have $\kappa^{ik}(x-x') \neq \kappa^{ik}(x'-x)$. Magnetic field obeys the induction equation:

$$\partial_t B^i + u^k B^i_k - B^k u^i_k = \eta \Delta B^i, \tag{3}$$

where lower indices denote derivatives with respect to the corresponding spatial coordinates, and we assume summation over the repeated indices. In the case of large magnetic Prandtl numbers (which is usually the case in astrophysical settings and is considered here), we neglect the magnetic diffusivity η. Using equation (3) as the Langevin equation for magnetic field fluctuations, one can easily derive the Fokker-Plank equation for the probability distribution function of the magnetic field measured at some point x, $P(B^i; x, t)$:

$$\partial_t P = \kappa_0 \Delta P + \kappa_2 B^2 \frac{\partial^2 P}{\partial (B^i)^2} - \frac{2}{d+1}\kappa_2 B^i B^k \frac{\partial^2 P}{\partial B^i \partial B^k} + 2\, g\, \varepsilon^{ikl} B^i \frac{\partial}{\partial B^k} \nabla_l P. \tag{4}$$

The coefficients in this equation can be read off from the following expansion of the velocity correlation function (2) in small argument $y \equiv x - x'$ for incompressible velocity field:

$$\kappa^{ik}(y) = \kappa_0 \delta^{ik} - \frac{\kappa_2}{2}\left(y^2 \delta^{ik} - \frac{2\, y^i y^k}{d+1}\right) + g\, \varepsilon^{ikl} y^l + \ldots, \tag{5}$$

where d is the space dimension ($d = 3$), and the g term describes kinetic helicity. Note that the value of g should obey the realizability condition

$$g^2 \leq \frac{5}{8}\kappa_0 \kappa_2, \tag{6}$$

that follows from the fact that $\langle (a\mathbf{u} + \nabla \times \mathbf{u})^2 \rangle \geq 0$ for any a.

For completeness, we would like to present here the main steps of the derivation of Eq. (4) from Eq. (3). First, let us introduce the so-called Z function defined as $Z(\lambda, x, t) = \exp(i\lambda^i B^i(x,t))$. Obviously, the average of this function over the random velocity field, u, or, equivalently, over the resulting magnetic field distribution $P(B; x, t)$, gives the Fourier transform of the distribution $P(B; x, t)$ with respect to B^i. Due to Eq. (3) the Z function satisfied the following differential equation:

$$\begin{aligned} \partial_t Z &= -i\lambda^i u^k B^i_k Z + i\lambda^i u^i_k B^k Z \\ &\equiv -u^k \frac{\partial}{\partial x^k} Z + u^i_k \lambda^i \frac{\partial}{\partial \lambda^k} Z. \end{aligned} \tag{7}$$

Let us now average this equation with respect to the random field u. To do this we first formally iterate the Eq. (7) once, getting:

$$\partial_t Z = u^k(t) \frac{\partial}{\partial x^k} \int\limits_{-\infty}^{t} \left[u^l(t') \frac{\partial}{\partial x^l} Z(t') - u^i_l(t') \lambda^i \frac{\partial}{\partial \lambda^l} Z(t') \right]$$
$$- u^i_k(t) \lambda^i \frac{\partial}{\partial \lambda^k} \int\limits_{-\infty}^{t} \left[u^l(t') \frac{\partial}{\partial x^l} Z(t') - u^l_j(t') \lambda^l \frac{\partial}{\partial \lambda^j} Z(t') \right], \tag{8}$$

where the integration is performed with respect to t' and we do not write the dependence on x in the right hand side of (8). The average over u can now be done *independently* of the Z function, since the Z function depends on the velocities taken at earlier times, $t' < t$, and due to causality, cannot depend on $u(t)$. We thus average u's and Z's independently in (8), and Fourier-transforming the resulting equation with respect to λ, arrive at Eq. (4).

Now, introducing new variables, the magnitude of the magnetic field B and its direction $n^i = B^i/B$ ($n^2 = 1$), we cast the Eq. (4) into the form:

$$\partial_t P = \kappa_0 \Delta P + \hat{L}_B P + \hat{L}_n P, \tag{9}$$

where

$$\hat{L}_B = \kappa_2 \frac{d-1}{d+1} \frac{1}{B^{d-1}} \frac{\partial}{\partial B} B^{d+1} \frac{\partial}{\partial B} \quad , \tag{10}$$

$$\hat{L}_n = \kappa_2 \left(\delta^{ik} - n^i n^k\right) \frac{\partial^2}{\partial n^i \partial n^k} - \kappa_2 (d-1) n^i \frac{\partial}{\partial n^i} + 2\, g\, \varepsilon^{ikl} n^i \frac{\partial}{\partial n^k} \nabla_l \,. \tag{11}$$

When differentiating with respect to n^i, we assume that all components of n^i are independent. This is a legitimate procedure since one can look for the solution in the form $P = \tilde{P}\delta(1-n^2)$, and observe that the δ function factors out, i.e., $\hat{L}_n\, \delta(1-n^2) f = \delta(1-n^2) \hat{L}_n f$ for an arbitrary function f.

The statistics of magnetic field magnitude and magnetic field direction are independent if they are initially independent. Let us thus look for the solution of the Fokker-Planck equation (4) in the factorized form

$$P = P_B(B,t)\, G(n,x,t), \tag{12}$$

where P_B is the function satisfying the equation

$$\partial_t P_B = \hat{L}_B P_B. \tag{13}$$

This equation can be solved exactly to give the well-known log-normal distribution of B (see e.g. [16]), but we do not need this solution for our present purposes. Let us concentrate on the equation for the G function

$$\partial_t G = \kappa_0 \Delta G + \hat{L}_n\, G\,. \tag{14}$$

If the kinetic helicity is nonzero, the last term in the operator (11) leads to coupling of n and x. A rigorous analysis would require solving the eigenvalue problem for the operator in the right hand side of (9), which we are going to do elsewhere. For now, we can accept that the G function in (12) corresponds to the *largest* eigenvalue of the operator in the right hand side of (14). The relevant structure of the solution can be simply understood in the following way. Let us first assume that G is independent of x. One can then formally expand G in powers of n^i:

$$G(n,t) = 1 + S^i n^i + Q^{ik} n^i n^k + D^{ikl} n^i n^k n^l + \dots, \tag{15}$$

where S, Q, and D are functions of t. Plugging this expansion in the Fokker-Planck equation (14), we easily check that the higher the order of the expansion coefficient, the faster it decays, and therefore for large times we can safely truncate the expansion (15) after the first relevant term, $G = 1 + S^i n^i$. Higher order terms can however be important for consideration of higher moments of the magnetic field but are irrelevant for our consideration of the mean magnetic field $\langle B^i \rangle$. This truncated solution, describing anisotropy of the magnetic field distribution, behaves as

$$S \propto \exp[-\kappa_2 (d-1) t]. \tag{16}$$

This suggests to look for the general solution of (14) in the form

$$G(n,x,t) = 1 + \bar{B}^i(x,t)\, n^i\, \exp[-\kappa_2(d-1)t]. \tag{17}$$

Equation (14) now reduces to (in three dimensions)

$$\partial_t \bar{B}^i = \kappa_0 \Delta \bar{B}^i + 2\, g\, \nabla \times \bar{B}^i, \tag{18}$$

and we recover the equation for the mean field (1), with the eddy diffusivity $\beta = \kappa_0$, and $\alpha = 2g$.

To reveal the physical sense of the function $\bar{B}^i(x,t)$, let us calculate the mean magnetic field $\langle B^i \rangle$ using the distribution (12). At first sight, it should decay since the magnetic field distribution function becomes more and more isotropic in the course of time, and the isotropic distribution has no preferred direction for the magnetic field B^i. However, the amplitude of the magnetic field grows making such an average nonvanishing. Indeed, multiplying the equation (13) by B^d (magnitude B times the volume factor B^{d-1}) and integrating with respect to B we get the average of the magnitude

$$\langle B \rangle = B_0 \exp[\kappa_2(d-1)t], \tag{19}$$

where we assumed $\langle B \rangle = B_0$ at the initial moment. Finally, averaging with respect to angles with the aid of the distribution G, we obtain $\langle B^i \rangle = \bar{B}^i(x,t)\, B_0/d$. Note that the exponential factors cancel out exactly in (16) and (19).

To conclude this section, we would like to explain the meaning of the averaging procedure in the above formulae. Mathematically, we average over the Gaussian ensemble (2). Physically, the averaging is understood to be performed over the space. More precisely, we average over the scales much larger than the velocity correlation length, but much smaller than the scales at which the function $\bar{B}^i(x,t)$ is changing. Since on these scales the velocity field is not correlated, we are effectively averaging over non-correlated regions of space, which is equivalent to the ensemble averaging. This is a sensible physical procedure, since, for example, the scale of the mean galactic magnetic field is $> 1\, kpc$, while the velocities are correlated at the scales $\leq 100\, pc$.

3. Equation (18) shows that the growth of the mean magnetic field is determined by the eigenvalues of the operator on the right hand side of (18). The solution depends on the magnitude of the helicity of the velocity field, on the geometry of the problem, and on the boundary conditions. Space Fourier transform of the Eq. (18) shows that the eigenvalues of the r.h.s. operator are: $\lambda_1 = -\kappa_0 k^2, \lambda_{2,3} = -\kappa_0 k^2 \pm 2gk$. Choosing g to be

positive, we obtain that the largest growth rate is equal to $\lambda_{\max} = g^2/\kappa_0$ and is achieved at $k = g/\kappa_0$ if this k is allowed in the system. Obviously, this growth can not proceed for infinitely long time, since eventually the growing magnetic field will be brought to equipartition with the velocity field. We therefore need to introduce the back reaction of the magnetic field to the velocity field. We do not know any rigorous way of including this reaction into the Eq. (4). A simple physical model can be suggested in the case when the mean magnetic field is much smaller than the magnetic fluctuations. This can happen when the following two conditions are met:

(1) The helicity of the velocity field is small, i.e., $\kappa_0/l, \kappa_2 l \gg g$, where l is the correlation length of the velocity field. In this case, the generated mean magnetic field is concentrated on the scales much larger than the velocity correlation length, and is growing with the rate much smaller than that of the fluctuations growth.

(2) The initial mean magnetic field is smaller than the initial magnetic fluctuations. The realizability condition (6) then ensures that the mean magnetic field is much smaller than the magnetic fluctuations at all consecutive times.

We now show that under these assumptions the equipartition is first achieved *locally* between the magnetic fluctuations and the velocity field. In a Lagrangian frame, magnetic field is rotated and stretched by the fluctuating velocity gradient matrix, u^i_k:

$$\frac{\mathrm{d}}{\mathrm{d}t} B^i = u^i_k B^k. \tag{20}$$

To model the back reaction, we want to represent this fluctuating gradient as some random part, $\tilde{u}^i_k$ plus some regular part (that does not include the random variable $\tilde{u}^i_k$) which is proportional to the magnetic fluctuations B^i. Field $\bar{B}^i$, being much smaller than the fluctuations, would give higher order terms in $g/\kappa_2 l$, gl/κ_0, and should not be relevant at least at the beginning of the saturation. We have the following traceless tensors at our disposal: $\varepsilon^{ikl}B^l$, B^i_k, and $T^{ik} \equiv B^i B^k - \delta^{ik}B^2/d$, which are constructed from the local magnetic field. The first two tensors can not be physically relevant, since they do not contribute to the energy exchange between the magnetic and velocity fields, which due to (20) is proportional to $\langle B^i B^k u^i_k \rangle$. As we show below, the choice of T^{ik} leaves the operator $\hat{L}_n$ intact and changes only the $\hat{L}_B$ part of the Fokker-Planck equation (4), (9). But before we proceed with the derivation we would like to give a more sound physical motivation for choosing the regular part of u^i_k in the form of T^{ik}, as suggested in [17].

Let us assume that the dynamo saturation starts when the viscous stress $\nu \Delta u^i$ in the right hand side of the incompressible MHD Navier-Stokes equation balances the dynamical stress

$$\nabla_k \left(B^i B^k - \frac{1}{2}\delta^{ik} B^2 \right) - \nabla_i\, p.$$

This balance seems to be reasonable at the *onset* of the back reaction, since it is the smallest fluid eddies that are less energetic and that are affected first by the growing magnetic field. Also, assuming that the pressure ensures the incompressibility of the velocity field (u^i_k must be traceless) we are left with

$$u^i_k = -\frac{1}{\nu} \left(B^i B^k - \frac{1}{d}\delta^{ik} B^2 \right) + \tilde{u}^i_k, \tag{21}$$

where the random part $\tilde{u}^i_k$ obeys (2), and the regular part is just $\frac{1}{\nu}T^{ik}$. The structure of the regular part of the velocity gradient shows that the magnetic field is stretched along itself, but not rotated. It is instructive to see the possible physical meaning of this ansatz. Numerical results of [18–20] suggest that the growing magnetic field becomes organized in filaments that occupy small fraction of space. The length of the filaments is of the order of the size of energy containing eddies. The magnetic field amplitude inside the filaments is large, and the saturation with the velocity fluctuations occurs first inside these filaments. With such saturation, small-scale velocity fluctuations cannot bend the filaments, since this would lead to a large restoring Lorentz force. The reduction of curvature of the filaments during the saturation process has indeed been observed numerically by Brandenburg *et al* [18]. We thus have to again exclude the local rotation (terms $\varepsilon^{ikl}B^l$ and B^i_k) and leave only the stretching along the magnetic field lines which is given by (21).

As we have mentioned, the ansatz (21) does not change the $\hat{L}_n$ part of the Fokker-Planck equation. The $\hat{L}_B$ part changes and takes the form:

$$\partial_t P_B = \hat{L}_B P_B + \frac{d-1}{\nu d} \frac{1}{B^{d-1}} \frac{\partial}{\partial B} B^{d+2} P_B. \tag{22}$$

The saturated PDF can be found analytically as a stationary solution of this equation, and turns out to be Gaussian,

$$P_B(B) = A \exp\left(-\frac{2}{3\nu\kappa_2} B^2 \right), \tag{23}$$

where A is the normalization constant and $d = 3$, as oppose to the log-normal distribution at the initial, kinematic, stage. Comparison with direct numerical simulations [17] shows that this gives the qualitatively correct behavior of the magnetic field PDF – the PDF indeed saturates at the Gaussian.[1] It is curious that the closure (21), suggested here for the onset of the back reaction, also works for the fully saturated regime. The

[1] A. Schekochihin was able to find the exact *time-dependent*

Gaussian stationary distribution of the magnetic field was earlier obtained in numerical simulations by Cattaneo [20] for the *central* part of the distribution function, while the *tails* of the PDF seemed to be non-Gaussian. We thus expect our local ansatz (21) to be valid only for the central part of the PDF; indeed, not being able to capture the spatial structure of the magnetic field it cannot account for the intermittency effects described by the non-Gaussian tails of the PDF. Note, that Gaussianity of the magnetic field distribution was also assumed in an analytical model for the mean magnetic field by Subramanian [21], see also numerics in [22].

The mechanism of the saturation of helical dynamo with small mean field is thus the following. The growing fluctuations of the magnetic field magnitude, described by $P_B(B,t)$, saturate *locally* with the velocity fluctuations, i.e., as described by the above model (21)-(22). After that the isotropisation of the magnetic field distribution proceeds faster than the amplitude growth, and suppresses further growth of $\langle B^i \rangle$: exponential decay of (16) is not compensated by (19) anymore, and due to the realizability condition (6), can not be compensated by the growth rate $\lambda_{\max}$ either. This provides a mechanism for the observed reduction of the α factor. Remarkably, our analytical result based on ansatz (21) is also in accord with the physical picture of dynamo saturation advocated by Cattaneo [19,20]: saturation occurs first locally in filaments that gradually fill the whole space.

Finally, it is instructive to see at what level the mean magnetic field should saturate. Due to (23), the saturated value of magnetic fluctuations energy is estimated as the energy value of the smallest turbulent eddies, $W \sim \nu\kappa_2 \sim E/Re^{1/2}$, where E is the total fluid energy and Re is the Reynolds number of the fluid turbulence. The saturation time can then be found from (19) to be $t_s \sim \log(W/W_0)/(4\kappa_2)$. Assuming that the mean magnetic field grows with the maximal possible growth rate, $\lambda_{\max} = g^2/\kappa_0$, we obtain for the saturated energy of the mean field, $\bar{W}$:

$$\bar{W} \sim \bar{W}_0 \, (W/W_0)^{\gamma} \, , \tag{24}$$

where $\gamma = g^2/(2\kappa_0\kappa_2)$. Setting $\bar{W}_0 \sim W_0$, and letting γ have the maximal possible value allowed by the realizability condition (6), $\gamma_m = 5/16$, we estimate:

$$\bar{W} \lesssim E \, (W_0/E)^{1-\gamma_m} \, Re^{-\gamma_m/2}. \tag{25}$$

We would like to stress the strong dependence of the saturated mean magnetic field on the value of the *initial* magnetic field, and rather weak dependence on the fluid Reynolds number. Since in our model the back reaction starts *before* the resistive scales are reached, the answer is independent of the magnetic diffusivity.

4. In the present paper we developed a framework for analyzing the generation and saturation of the mean magnetic field, which is based on the Kazantsev-Kraichnan ensemble for the velocity field. Supplemented by the physically motivated ansatz for the magnetic field back reaction, such a model accounts for the growing mean magnetic field exactly.

Let us now discuss possible limitations of the approach. The solution of the mean-field equation requires the specification of the particular geometry and can be different for different boundary conditions. Of course, the condition of homogeneity of the velocity fluctuations could be incompatible with real boundary conditions, which would force us to consider more general velocity correlators, $\kappa(x+x';x-x')$ and to solve the eigenvalue problem analogous to (14). Another generalization may be necessary for the case of arbitrary, not small, magnitude of the mean magnetic field. In this case, the ansatz for the regular part of the velocity field can include large $\bar{B}^i$ field as well. Also, the neglected tensor B^i_k, though not leading to the energy transfer, can be important for the isotropisation of the PDF in this case. Another possible generalization is to consider a compressible velocity field. All these questions are for the future.

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solution of Eq. (22), which allowed him to observe the evolution of $P_B(B,t)$ from the kinematic log-normal to the saturated Gaussian form. (Private communication, 2001.)

[1] D. Lynden-Bell (ed.), *Cosmical Magnetism* (Kluwer Academic Publishers, Dordrecht 1994).

[2] E. N. Parker, *Cosmical Magnetic Fields* (Clarendon Press, Oxford 1979).

[3] R. H. Kraichnan and S. Nagarajan, Phys. Fluids **10**, (1967) 859.

[4] A. P. Kazantsev, Sov. Phys. JETP **26** (1968) 1031.

[5] R. M. Kulsrud and S. W. Anderson, ApJ **396** (1992) 606.

[6] A. Gruzinov, S. Cowley, and R. Sudan, Phys. Rev. Lett **77** (1996) 4342.

[7] M. Chertkov, G. Falkovich, I. Kolokolov, and M. Vergassola, Phys. Rev. Lett. **83** (1999) 4065.

[8] H. K. Moffatt, *Magnetic Field Generation in Electrically Conducting Fluids* (Cambridge University Press, Cambridge 1978).

[9] S. I. Vainshtein, Phys. Rev. Lett. **80** (1998) 4879.
[10] D. Biskamp, *Nonlnear Magnetohydrodynamics*, (Cambridge University Press, 1997).
[11] A. V. Gruzinov and P. H. Diamond, Phys. Rev. Lett. **72** (1994) 1651; A. V. Gruzinov and P. H. Diamond, Physics of Plasmas **2** (1995) 1941.
[12] E. G. Blackman and G. B. Field, ApJ **521** (1999) 597.
[13] S. I. Vainshtein and F. Cattaneo, ApJ **393** (1992) 199.
[14] A. Brandenburg, ApJ **550** (2001) 824.
[15] E. G. Blackman and G. B. Field, astro-ph/0012048.
[16] S. Boldyrev and A. Schekochihin, Phys. Rev. E **62** (2000) 545.
[17] S. Boldyrev and A. Brandenburg, (2000) unpublished.
[18] A. Brandenburg, *et al* J. Fluid Mechanics **306** (1996) 325.
[19] F. Cattaneo, in F. P. Pijpers et al. (eds.) *SCORe'96: Solar Convection and Oscillations and their Relationship* (Kluwer Academic Publishers, Netherlands 1997) 201-222;
[20] F. Cattaneo, in A. Hanslmeier and M. Messerotti (eds.) *Motions in the Solar Atmosphere,* (Kluwer Academic Publishers, Netherlands 1999) 119-137.
[21] K. Subramanian, Phys. Rev. Lett. **83** (1999) 2957.
[22] A. Brandenburg and K. Subramanian, Astron. Astrophys. **361** (2000) L33-L36.